11.1 HW
1-10, 15, 19, 21, 29, 35, 37, 45, 53, 57, 63, 67, 75, 79, 83

s given

0

able 4. ent on on No- ent on

trip to

in the

loans. to be

im is

$B.

30

of

11.1 Assess Your Understanding

'Are You Prepared?'

Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

1. For the function $f(x) = \frac{x-1}{x}$, find $f(2)$ and $f(3)$. (pp. 61–63)

2. *True or False:* A function is a relation between two sets D and R so that each element x in the first set D is related to exactly one element y in the second set R. (pp. 56–61)

3. If \$1000 is invested at 4% per annum compounded semi-annually, how much is in the account after 2 years? (pp. 315–322)

4. How much do you need to invest now at 5% per annum compounded monthly so that in 1 year you will have \$10,000? (pp. 315–322)

Concepts and Vocabulary

5. A(n) ______ is a function whose domain is the set of positive integers.

6. For the sequence $\{s_n\} = \{4n - 1\}$, the first term is $s_1 =$ ______ and the fourth term is $s_4 =$ ______.

7. $\sum_{k=1}^{4}(2k) =$ ______.

8. *True or False:* Sequences are sometimes defined recursively.

9. *True or False:* A sequence is a function.

10. *True or False:* $\sum_{k=1}^{2} k = 3$

Skill Building

In Problems 11–16, evaluate each factorial expression. Verify your results using a graphing utility.

11. $10!$ 12. $9!$ 13. $\frac{9!}{6!}$ 14. $\frac{12!}{10!}$ 15. $\frac{3!7!}{4!}$ 16. $\frac{5!8!}{3!}$

In Problems 17–28, write down the first five terms of each sequence.

17. $\{n\}$
18. $\{n^2 + 1\}$
19. $\left\{\frac{n}{n+2}\right\}$
20. $\left\{\frac{2n+1}{2n}\right\}$
21. $\{(-1)^{n+1}n^2\}$
22. $\left\{(-1)^{n-1}\left(\frac{n}{2n-1}\right)\right\}$
23. $\left\{\frac{2^n}{3^n+1}\right\}$
24. $\left\{\left(\frac{4}{3}\right)^n\right\}$
25. $\left\{\frac{(-1)^n}{(n+1)(n+2)}\right\}$
26. $\left\{\frac{3^n}{n}\right\}$
27. $\left\{\frac{n}{e^n}\right\}$
28. $\left\{\frac{n^2}{2^n}\right\}$

In Problems 29–36, the given pattern continues. Write down the nth term of each sequence suggested by the pattern.

29. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$
30. $\frac{1}{1\cdot 2}, \frac{1}{2\cdot 3}, \frac{1}{3\cdot 4}, \frac{1}{4\cdot 5}, \ldots$
31. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$
32. $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \ldots$
33. $1, -1, 1, -1, 1, -1, \ldots$
34. $1, \frac{1}{2}, 3, \frac{1}{4}, 5, \frac{1}{6}, 7, \frac{1}{8}, \ldots$
35. $1, -2, 3, -4, 5, -6, \ldots$
36. $2, -4, 6, -8, 10, \ldots$

In Problems 37–50, a sequence is defined recursively. Write the first five terms.

37. $a_1 = 2;\ a_n = 3 + a_{n-1}$
38. $a_1 = 3;\ a_n = 4 - a_{n-1}$
39. $a_1 = -2;\ a_n = n + a_{n-1}$
40. $a_1 = 1;\ a_n = n - a_{n-1}$
41. $a_1 = 5;\ a_n = 2a_{n-1}$
42. $a_1 = 2;\ a_n = -a_{n-1}$
43. $a_1 = 3;\ a_n = \frac{a_{n-1}}{n}$
44. $a_1 = -2;\ a_n = n + 3a_{n-1}$
45. $a_1 = 1;\ a_2 = 2;\ a_n = a_{n-1} \cdot a_{n-2}$
46. $a_1 = -1;\ a_2 = 1;\ a_n = a_{n-2} + na_{n-1}$
47. $a_1 = A;\ a_n = a_{n-1} + d$
48. $a_1 = A;\ a_n = ra_{n-1},\ r \neq 0$
49. $a_1 = \sqrt{2};\ a_n = \sqrt{2 + a_{n-1}}$
50. $a_1 = \sqrt{2};\ a_n = \sqrt{\frac{a_{n-1}}{2}}$

In Problems 51–60, write out each sum.

51. $\sum_{k=1}^{n}(k+2)$
52. $\sum_{k=1}^{n}(2k+1)$
53. $\sum_{k=1}^{n}\frac{k^2}{2}$
54. $\sum_{k=1}^{n}(k+1)^2$
55. $\sum_{k=0}^{n}\frac{1}{3^k}$

56. $\sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$ **57.** $\sum_{k=0}^{n-1} \frac{1}{3^{k+1}}$ **58.** $\sum_{k=0}^{n-1} (2k+1)$ **59.** $\sum_{k=2}^{n} (-1)^k \ln k$ **60.** $\sum_{k=3}^{n} (-1)^{k+1} 2^k$

In Problems 61–70, express each sum using summation notation.

61. $1 + 2 + 3 + \cdots + 20$

62. $1^3 + 2^3 + 3^3 + \cdots + 8^3$

63. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{13}{13+1}$

64. $1 + 3 + 5 + 7 + \cdots + [2(12) - 1]$

65. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots + (-1)^6\left(\frac{1}{3^6}\right)$

66. $\frac{2}{3} - \frac{4}{9} + \frac{8}{27} - \cdots + (-1)^{11+1}\left(\frac{2}{3}\right)^{11}$

67. $3 + \frac{3^2}{2} + \frac{3^3}{3} + \cdots + \frac{3^n}{n}$

68. $\frac{1}{e} + \frac{2}{e^2} + \frac{3}{e^3} + \cdots + \frac{n}{e^n}$

69. $a + (a + d) + (a + 2d) + \cdots + (a + nd)$

70. $a + ar + ar^2 + \cdots + ar^{n-1}$

In Problems 71–82, find the sum of each sequence (a) algebraically and (b) using a graphing utility.

71. $\sum_{k=1}^{10} 5$ **72.** $\sum_{k=1}^{20} 8$ **73.** $\sum_{k=1}^{6} k$ **74.** $\sum_{k=1}^{4} (-k)$

75. $\sum_{k=1}^{5} (5k + 3)$ **76.** $\sum_{k=1}^{6} (3k - 7)$ **77.** $\sum_{k=1}^{3} (k^2 + 4)$ **78.** $\sum_{k=0}^{4} (k^2 - 4)$

79. $\sum_{k=1}^{6} (-1)^k 2^k$ **80.** $\sum_{k=1}^{4} (-1)^k 3^k$ **81.** $\sum_{k=1}^{4} (k^3 - 1)$ **82.** $\sum_{k=0}^{3} (k^3 + 2)$

Applications and Extensions

83. **Credit Card Debt** John has a balance of \$3000 on his Discover card that charges 1% interest per month on any unpaid balance. John can afford to pay \$100 toward the balance each month. His balance each month after making a \$100 payment is given by the recursively defined sequence

$$B_0 = \$3000, \qquad B_n = 1.01B_{n-1} - 100$$

(a) Determine John's balance after making the first payment. That is, determine B_1.
(b) Using a graphing utility, determine when John's balance will be below \$2000. How many payments of \$100 have been made?
(c) Using a graphing utility, determine when John will pay off the balance. What is the total of all the payments?
(d) What was John's interest expense?

84. Car Loans Phil bought a car by taking out a loan for \$18,500 at 0.5% interest per month. Phil's normal monthly payment is \$434.47 per month, but he decides that he can afford to pay \$100 extra toward the balance each month. His balance each month is given by the recursively defined sequence

$$B_0 = \$18{,}500, \qquad B_n = 1.005B_{n-1} - 534.47$$

(a) Determine Phil's balance after making the first payment. That is, determine B_1.
(b) Using a graphing utility, determine when Phil's balance will be below \$10,000. How many payments of \$534.47 have been made?
(c) Using a graphing utility, determine when Phil will pay off the balance. What is the total of all the payments?
(d) What was Phil's interest expense?

85. Trout Population A pond currently has 2000 trout in it. A fish hatchery decides to add an additional 20 trout each month. In addition, it is known that the trout population is growing 3% per month. The size of the population after n months is given by the recursively defined sequence

$$p_0 = 2000, \qquad p_n = 1.03p_{n-1} + 20$$

(a) How many trout are in the pond at the end of the second month? That is, what is p_2?
(b) Using a graphing utility, determine how long it will be before the trout population reaches 5000.

86. Environmental Control The Environmental Protection Agency (EPA) determines that Maple Lake has 250 tons of pollutants as a result of industrial waste and that 10% of the pollutant present is neutralized by solar oxidation every year. The EPA imposes new pollution control laws that result in 15 tons of new pollutant entering the lake each year. The amount of pollutant in the lake at the end of each year is given by the recursively defined sequence

$$p_0 = 250, \qquad p_n = 0.9p_{n-1} + 15$$

(a) Determine the amount of pollutant in the lake at the end of the second year. That is, determine p_2.
(b) Using a graphing utility, provide pollutant amounts for the next 20 years.
(c) What is the equilibrium level of pollution in Maple Lake? That is, what is $\lim_{n\to\infty} p_n$?

87. Roth IRA On January 1, 1999, Bob decided to place \$500 at the end of each quarter into a Roth Individual Retirement Account.